## LECTURE 15 TO 17- DIRECTION CONTROL VAVLES

## SELF EVALUATION QUESTIONS AND ANSWERS

1. A spool valve is controlling an ideal motor. The system characteristics are given in the following table. Calculate the following (a) Load pressure (b) Load flow (c) Power delivered by the motor (d) Motor displacement


| Parameters | Value |
| :--- | :---: |
| Orifice discharge constant $\left(C_{d}\right)$ | 0.62 |
| Valve area gradient $(\mathrm{w})$ | 0.01 m |
| Valve opening $\left(x_{v}\right)$ | 0.25 mm |
| Supply side pressure $\left(p_{s}\right)$ | 20 MPa |
| Drain side pressure $\left(p_{d}\right)$ | 0 |
| Upstream pressure to $\operatorname{motor}\left(p_{1}\right)$ | 12 MPa |
| Fluid density | $855 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}}$ |
| Motor speed $(\mathrm{n})$ | 1000 RPM |

2. A cylinder with bore diameter of 8 cm and a rod diameter of 4 cm is to be used in a system with a 30 LPM pump. Use the graph in the figure to determine the pressure drops across the DCV when the cylinder is retracting ( $P->B, A->T$ ).

3. A cylinder with bore diameter of 10 cm and a rod diameter 0 cm is to be used in a system with a 200 LPM pump. Use the graph in the figure to determine the pressure drops across the DCV when the cylinder is retracting ( $\mathrm{P}->\mathrm{B}, \mathrm{A}->\mathrm{T}$ ).


## Q1 Solution

First calculate certain properties associated with the initial conditions. Calculate the load pressure. The valve was indicated to be ideal with no leakage and the motor is also ideal. Thus:
$\mathrm{p}_{2}=\mathrm{p}_{\mathrm{s}}-\mathrm{p}_{1}=20 \times 10^{6}-12 \times 10^{6}=8 \times 10^{6} \mathrm{~Pa}=8 \mathrm{MPa}$

So the load pressure can be determined:
$\mathrm{p}_{\mathrm{L}}=\mathrm{p}_{1}-\mathrm{p}_{2}=12 \times 10^{6}-8 \times 10^{6}=4 \times 10^{6} \mathrm{~Pa}=4 \mathrm{MPa}$

Calculate the initial load flow:
$\mathrm{Q}_{\mathrm{L}}=\mathrm{C}_{\mathrm{d}} \mathrm{WX}_{\mathrm{v}} \sqrt{\frac{\left(\mathrm{p}_{\mathrm{s}}-\mathrm{p}_{\mathrm{L}}\right)}{\rho}}$
$=0.62 \times 0.01 \times 0.25 \times 10^{-3} \sqrt{\frac{\left(20 \times 10^{6}-4 \times 10^{6}\right)}{855}}$
$=0.212 \times 10^{-3} \mathrm{~m}^{3} / \mathrm{s}$

Now we shall evaluate $K_{q} K_{c}$ and $K_{p}$ at initial operating point. The flow gain coefficent, $K_{q}$
$\mathrm{K}_{\mathrm{q}}=C_{d} w \sqrt{\frac{\left(p_{s}-p_{L}\right)}{\rho}}$

$$
\begin{aligned}
& =0.62 \times 0.01 \sqrt{\frac{\left(20 \times 10^{6}-4 \times 10^{6}\right)}{855}} \\
& =0.8481 \mathrm{~m}^{2} / \mathrm{s}
\end{aligned}
$$

The flow pressure coefficeint, $\mathrm{K}_{\mathrm{c}}$

$$
\mathrm{K}_{\mathrm{c}}=\mathrm{K}_{\mathrm{q}} \frac{x_{v}}{2\left(p_{s}-p_{L}\right)}
$$

$$
=0.8481 \frac{0.25 \times 10^{-3}}{2\left(20 \times 10^{6}-4 \times 10^{6}\right)}
$$

$$
=6.626 \times 10^{-12} \mathrm{~m}^{3} / \mathrm{s} . \mathrm{Pa}
$$

The pressure sensitivity coefficeint,

$$
\mathrm{K}_{\mathrm{p}}=\frac{K_{q}}{K_{c}}=\frac{0.8481}{6.626 \times 10^{-12}}=0.128 \times 10^{12} \mathrm{~Pa} / \mathrm{m}
$$

Calculate the power delivered by the motor

$$
P=\mathrm{p}_{\mathrm{L}} \mathrm{Q}_{\mathrm{L}}=4 \times 10^{6} \times 0.212 \times 10^{-3}=848.1 \text { watts }
$$

Calcute the motor shaft speed , $\omega$ :
$\omega: \frac{2 \pi n}{60}=\frac{2 x \pi x 1000}{60}=104.7 \mathrm{rad} / \mathrm{s}$

Calculate the power coefficeint $\mathrm{K}_{\mathrm{pwr}}=\frac{\mathrm{P}}{\omega^{2}}=\frac{848.1}{104.7^{2}}=77.34 \times 10^{-3} \mathrm{~W} / \mathrm{rad}^{2}$

Calculate th motor displacement $D_{m}$
$\mathrm{D}_{\mathrm{m}}=\frac{\mathrm{Q}_{\mathrm{L}}}{\omega}=\frac{0.212 \times 10^{-3}}{104.7^{2}}=2.025 \times 10^{-6} \mathrm{~m}^{-3} / \mathrm{rad}$

Now we shall develop a relationship that will allow the calculation of load pressure $p_{L}$, when the load
follows $\mathrm{P}=\mathrm{K}_{\mathrm{pwr}} \omega^{2}=\mathrm{K}_{\mathrm{pwr}}\left(\frac{\mathrm{Q}_{\mathrm{L}}}{\mathrm{D}_{\mathrm{m}}}\right)^{2}$

Which leads to an expression for $Q_{L}$
$\mathrm{Q}_{\mathrm{L}}=\left(\frac{\mathrm{D}_{\mathrm{m}}{ }^{2}}{\mathrm{~K}_{\mathrm{pwr}}}\right) p_{L}$

Noting that $\mathrm{Q}_{\mathrm{L}}$ may also be expressed as
$\mathrm{Q}_{\mathrm{L}}=w x_{\mathrm{V}} C_{d} \sqrt{\frac{\left(p_{s}-p_{L}\right)}{\rho}}$

Thus $\mathrm{Q}_{\mathrm{L}}$ can be eliminated leaving
$\mathrm{Q}_{\mathrm{L}}=\left(\frac{\mathrm{D}_{\mathrm{m}}{ }^{2}}{\mathrm{~K}_{\mathrm{pwr}}}\right) p_{L}=w x_{\mathrm{v}} C_{d} \sqrt{\frac{\left(p_{s}-p_{L}\right)}{\rho}}$

This expression can be rearranged as a quadratic equation to calucalte $\mathrm{p}_{\mathrm{L}}$
$\mathrm{p}_{\mathrm{L}}^{2}+\left(\frac{w x_{\mathrm{v}} C_{d} \mathrm{~K}_{\mathrm{pwr}}}{\mathrm{D}_{\mathrm{m}}{ }^{2}}\right) \frac{1}{\rho} p_{L}-\left(\frac{w x_{\mathrm{v}} C_{d} \mathrm{~K}_{\mathrm{pwr}}}{\mathrm{D}_{\mathrm{m}}{ }^{2}}\right) \frac{1}{\rho} p_{s}=0$

Let
$\mathrm{C}=\left(\frac{w x_{\mathrm{v}} C_{d} \mathrm{~K}_{\mathrm{pwr}}}{\mathrm{D}_{\mathrm{m}}{ }^{2}}\right)^{2} \frac{1}{\rho}$

For the new operating condition where
$x_{\mathrm{v}}=1.05 x_{\mathrm{v}}=1.05 x 0.25 \times 10^{-3}=0.2625 \times 10^{-3} \mathrm{~m}$
$C=\left(\frac{0.62 \times 0.01 \times 0.2625 \times 10^{-3} \times 77.34 \times 10^{-3}}{\left(2.025 \times 10^{-6}\right)^{2}}\right)^{2} \frac{1}{855}=1.103 \times 10^{6}$

Now solve the quadratic in $\mathrm{p}_{\mathrm{L}}$ :
$\mathrm{p}_{\mathrm{L}}^{2}+C \mathrm{p}_{\mathrm{L}}-C \mathrm{p}_{\mathrm{s}}=0$

Only the positive root has meaning in this context, so substituting $\mathrm{C}=11.103 \times 10^{6}$
$\mathrm{p}_{\mathrm{L}}=\frac{-1.103 \times 10^{6} \pm \sqrt{\left(1.103 \times 10^{6}\right)^{2}+4 \times 1.103 \times 10^{6} \times 20 \times 10^{6}}}{2}=4.177 \times 10^{6} \mathrm{~Pa}=4.177 \mathrm{MPa}$

Calcute the load flow, $\mathrm{Q}_{\mathrm{L}}$ under the new conditions:
$\mathrm{Q}_{\mathrm{L}}=w x_{\mathrm{V}} C_{d} \sqrt{\frac{\left(p_{s}-p_{L}\right)}{\rho}}=0.62 x 0.01 \times 0.2625 \times 10^{-3} \sqrt{\frac{\left(20 \times 10^{6}-4.177 \times 10^{6}\right)}{855}}=0.2214 \times 10^{-3} \mathrm{~m}^{3} / \mathrm{s}$

Now caluclate $\omega$ and verify that the predicted $\mathrm{p}_{\mathrm{L}}$ is correct by calculating the power under the new conditons in two separate ways:
$\omega=\frac{\mathrm{Q}_{\mathrm{L}}}{\mathrm{D}_{\mathrm{m}}}=\frac{0.2214 \times 10^{-3}}{2.025 \times 10^{-6}}=109.4 \mathrm{rad} / \mathrm{s}$
calculate the power from
$P=\mathrm{p}_{\mathrm{L}} \mathrm{Q}_{\mathrm{L}}=4.177 \times 10^{6} \times 0.221 \times 10^{-3}=924.8$ watts
and from:
$\mathrm{P}=\mathrm{K}_{\mathrm{pwr}} \omega^{2}=\mathrm{K}_{\mathrm{pwr}}\left(\frac{\mathrm{Q}_{\mathrm{L}}}{\mathrm{D}_{\mathrm{m}}}\right)^{2}=77.34 \times 10^{-3} \times 109.4^{2}=924.8$ watts

Both the power expressions yield the same result. Calculate the increment in valve opening
$\Delta x_{v}=0.05 \times 0.25 \times 10^{-3}=0.0125 \times 10^{-3} \mathrm{~m}$

Now calculate the approximate value of the load flow, $\mathrm{Q}_{\mathrm{L}}$, using $K_{\mathrm{q}}$ from the linearisation appraoch:

$$
\mathrm{Q}_{\mathrm{L}} \approx \mathrm{Q}_{\mathrm{L}}(\mathrm{old})+\Delta x_{v} K_{\mathrm{q}}=0.2120 \times 10^{-3}+0.0125 \times 10^{-3} \times 0.8481=0.2214 \times 10^{-3} \mathrm{~m}^{3} / \mathrm{s}
$$

and the approximate value of the load pressure $\mathrm{p}_{\mathrm{L}}$, using $K_{\mathrm{q}}$
$\mathrm{p}_{\mathrm{L}} \approx \mathrm{p}_{\mathrm{L}}(\mathrm{old})+\Delta x_{v} K_{\mathrm{p}}=4 \times 10^{6}+0.0125 \times 0.128 \times 10^{12}=5.6 \times 10^{6} \mathrm{~Pa}$

## Q2 Solution:

The flow from $P$ to $B$ is the pump flow into the rod end, so this can be read from the graph

$$
\Delta p=1.5 \text { bar (approx.) }
$$

The flow from A->T is the return flow out of the blind end. This flow rate is greater than the pump flow and must be determined by the following method
a. Calculate the piston area

$$
A_{p=} \frac{\pi}{4}\left(D_{P}^{2}\right)=\frac{\pi}{4}\left(8^{2}\right)=50.264 \mathrm{~cm}^{2}
$$

b. Calculate the rod area

$$
A_{R}=\frac{\pi}{4}\left(D_{R}^{2}\right)=\frac{\pi}{4}\left(4^{2}\right)=12.566 \mathrm{~cm}^{2}
$$

c. Calculate the return flow

$$
Q_{\text {return }, R}=\frac{Q_{\text {pump }}}{A_{p}-A_{R}} x A_{p}=\frac{Q_{\text {pump }}}{50.264-12.566} \times 50.264=1.33 \times 30=40 \mathrm{LPM}
$$

The flow from A to $T$ can now be read from the graph

$$
\Delta \mathrm{p}=5.8 \text { (approx.) }
$$



## Q3 Solution:

The flow from $P$ to $B$ is the pump flow into the rod end, so this can be read from the graph

$$
\Delta \mathrm{p}=1.5 \text { bar (approx.) }
$$

The flow from A->T is the return flow out of the blind end. This flow rate is greater than the pump flow and must be determined by the following method
a. Calculate the piston area

$$
A_{p=}=\frac{\pi}{4}\left(D_{P}^{2}\right)=\frac{\pi}{4}\left(10^{2}\right)=78.73 \mathrm{~cm}^{2}
$$

b. Calculate the rod area

$$
A_{R}=\frac{\pi}{4}\left(D_{R}^{2}\right)=\frac{\pi}{4}\left(6^{2}\right)=28.3 \mathrm{~cm}^{2}
$$

c. Calculate the return flow

$$
Q_{\text {return }, R}=\frac{Q_{\text {pump }}}{A_{p}-A_{R}} x A_{p}=\frac{Q_{\text {pump }}}{78.73-28.3} x 78.73=1.576 \times 200=315.36 \mathrm{LPM}
$$

The flow from A to $T$ can now be read from the graph $\quad \Delta p=1.9$ bar (approx.)

